K-FACTORS FOR COLUMN IN BRACED AND UNBRACED FLAT PLATE SYSTEM

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ABSTRACT
Two approaches between (K-factor approximate approach), and (Column stiffness modification approach) are presented for calculating the effective length factor (K-factor) of columns in braced and unbraced flat plate system. This study integrates K-factor, which is calculated from theoretical values based on records obtained from measurements between the stiffness coefficients derived by Notes of ACI-318-12 code [1] contained within the equivalent frame method (EFM). A subassembly is used to add second-order analysis (P-Δ) effects, and column stability functions are used to explain differences in computational accuracy between two methods for incorporating stiffness at the column-slab joints. The second method proceeds through alignment and is developed using high-accuracy approximate equations to the current US National codes provisions with a refined calculation method performed through that. The calculations consist of solving a number of problems and comparing the results obtained using both proposed methods—based on Newton-Raphson iterations—to those obtained with the approximate equations. Comparisons are made with the proposed method, practical alignment chart methods and so on for different world specifications. The predicted results show the reliability of the proposed equations as well as simplicity of enhanced alignment chart method that can be easily incorporated to practical engineering use.

Keywords: K-factor, flat plate, stability functions, stiffness, braced frame, unbraced frame.

1. Introduction

The K-factor calculation for columns has gained popularity, and its methods have expanded through the incorporation of different assumptions which aims to provide simplified complex procedures into practical, applicable strategies. Many international codes and standards have included the K-factor calculation since it plays an important role in sophisticated structural assessments. This factor is not constant but varies proportionate to the structure's geometry, material properties, end-restraint conditions etc. Prior studies have spent multiple work in developing and enhancing the equivalent frame method. To place this work in context, a summary of studies relevant to the current research is given below.

Corley et al. EFM [2] was introduced by modelling the construction and functional components in a 2D frame interconnected system which influences code-based design standards, enabling designers to practically examine moments, shear forces and deformations developed within slab systems in practice. Corley and Jirsa [3] investigated a simplified analytical method, which represented complex slab systems as equivalent planar frames for source computational purposes. They emphasized the importance of accounting for factors such as torsional stiffness, load distribution, and boundary conditions to ensure realistic results. The study provided critical insights into the limitations of traditional (EFM) assumptions and proposed adjustments to enhance its reliability, particularly in irregular or non-uniform slab configurations

By benefit from the slab-beam and column stiffness data outline in tables A1 and A7 of appendix 20A presented in ACI-318-12 [1], these stiffness data are fitted to obtain its corresponding stiffness and stability functions equations, the fitted equations play a major role in the current study. To include the effect of axial force on column bending stiffness, a modified stiffness method was proposed by Al-Ghalibi and Al-Humeidawi [4] investigated the importance of the calculation of column stiffness within (EFM). The research aimed to address limitations in traditional (EFM) practices, particularly in accurately determining column stiffness, the authors sought to refine the accuracy and reliability of column stiffness calculations.

Among the most significant prior studies that adopted the calculation of (K-factor) for columns—relying on various hypotheses, methodologies, and scenarios to achieve structural behavior closely approximating real-world conditions—are the following:

The alignment charts of the current US National codes which used the original G parameter are adopted in the present study were introduced by Julian and Lawrence [5] they presented a graphical method using nomograms to simplify the analysis of column stability in continuous structural frames. Developed nomograms—a visual

computational tool—accounted for the interaction between columns and surrounding structural elements, enabling engineers to rapidly estimate K-factors without complex calculations.

LeMessurie and William [6] made some of better assumptions than Julian and Lawrence [5] by developing a more simpler but physically sound method for P-Delta effect in the frames. That is followed by the limitations of standard first-order methods that neglect the contribution from axial loads acting on axial displacements. This is a computational design method that can take into consideration geometric nonlinearity, providing engineers with the ability to predict frame behaviour subjected to load without going through complex iterative analyses.

Yura [7] examined the K-factors for unbraced frames indicating that lateral stability depends on the inherent stiffness of frame, not bracing provided by attachments. This involves investigating how columns interact with other elements of the frame, including consideration of joint rigidity and load distribution within an arbitrary geometry.

Duan and Chen [8] investigated K-factors in braced frames; their study included assessing the reliability of existing Effective Length Factor (K-factor) estimation methods, specifically by examining braced frames that obtain lateral stability primarily through the use of internal or external bracing. Abstract: Improved methodologies for calculating the Effective Length Factor that overcome limitations inherent in traditional approaches are proposed.

Alkraidi, et. al. Using high-density polyethylene fibres derived from used plastic cans to improve the mechanical properties of concrete [9]. The results indicated that the incorporation of 6% fibers improved compressive strength from 37.4 MPa to 49.3 MPa, tensile strength from2.9MPato5.2MPaandflexural strengthfrom3.5MPatonewtothevalueof9.9Mpa[1]. The study evaluates and concludes that such fibres are long lasting and effective way to increase performance of concrete.

The work of Duan and Chen [10] was looking at the assessment of the accuracy of existing simplified methods in determining the Effective Length Factor for unbraced frames. It was the limitations of traditional approaches which ledthe authors to propose improved methodologies towards calculating the Effective Length Factor.

Yura et al. [11] Investigate the determination of effective column lengths in unbraced structural frames This study for estimating K has faced objection due to the ignorancy about accurately calculating effective length factor (K) on wide range of column structures where stability and load bearing capacity are predominant design parameters using a large set of experimental data. The authors suggest a methodology that considers neighboring members and behavior of frames, which is more accurate than traditional methods.

Dissertation at University of California, BerkeleyLui [12] provides a new analytical method for determining the length factor (K), while the study criticises conventional process design charts which frequently depend on simplified boundary conditions and ignore significant inelastic action moments and variable frame bends. A generalized analytical framework that combines both column and frame deformations along with ge-ometric and material nonlinearities which can better capture buckling behavior, is formulated in [11].

More recently, Dumonteil [13], introduced simplified equations to compute the Effective Length Factors of structural columns. The advantage of the research was to provide engineers with usable and easy-going formulas that could be used in place of more complex methods that would take significant time. Develops equations to help study column behavior under different boundary conditions that provide a compromise between accuracy and simplicity[28].

Chen et al. Compared the K-Factors for framed columns as specified in ACI Building Code[14] Abstract—This research looks to assess the efficacy and accuracy of existing methods for calculating the Effective Length Factor (K) in structural column design. The authors outlined some suggestions to enhance current methods, particularly through the impact of boundary conditions and interelements interaction.

Chen et al. Yamane et al. [15],[29] examined K-factors for framed columns restrained by tapered girders in accordance with US design codes. Abstract: The work presented in this paper was based on analytical study of tapered girders influence on the stability and behaviour of frame columns. They assessed the accuracy of current design provisions and determined potential limitations in their applications to structures with tapered girders.

A historical approach of the development of K-factor equations was presented by dumonteil [16]; this study followed an evolution from initial phenomenological formulations into proposals with theoretically-based approaches to model the temperature dependence of permeability. Dumonteil said following the historical trail to these equations helps provide a proper view of their uses and limits today.

Abdulabbas and Alasadi [17],[25] researched sustainable concrete and metal waste brittle fibers, it is possible to improve strength and elasticity with regard to bottle cap, whereas the can wall has decreased them. The study also proved the applicability of CEB-FIP Model Code 2010 in predicting dynamic properties of green concrete. The results obtained from the study confirm the utility of industrial waste within sustainable construction making it a potentially viable way to reuse current waste stream (below 300 µm fractions) into new high-quality and smart products.

Al-Ghalibi [18],[26]study focused on the way the deformation of girders positioned on elastic foundations, like variations of soil or other deformable bases can alter the stability of his columns. These results highlight the need to account for elastic foundation effects, as they may lead to either a conservative or non-conservative estimate of column stability.

This study focuses on calculating the effective length factor for columns in both braced and unbraced flat plate building systems. The column stiffness coefficients specified in ACI-318-12 [1] ,[27]were adopted and further

developed to account for second-order stability analysis (P-Δ effects). Two solution methods (K-factor approximate approach and column stiffness modification approach) are explored for both braced and unbraced frames. Newton-Raphson iteration is used for closed-form solutions and then approximate K-factor equations are proposed. The study extended to be simpler and practical by suggested method of using the alignment charts of the current US national codes by developing the original G factor using stiffness parameters.

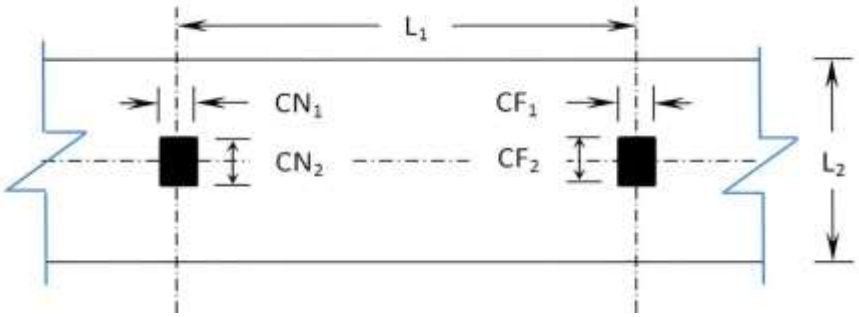
The derivation of K-factor is based on calculation assumptions; these assumptions are simplified the modeling using subassembly model. In this study, the following assumptions are used:

a. Columns buckle simultaneously.
b. Materials are linear elastic and all members have constant cross-section.
c. Slab-beam portions are subjected zero axial force.
d. All joints are rigid.
e. Upper and lower story are of equal slab thickness.
f. The axial force parameter of all columns is equal.
g. For braced frames, slab-beam portions are bent in single curvature.

2. For unbraced frames, slab-beam portions are bent in double curvature.Slab-beam stiffness

For flat plate system, the slop-deflection equations of slab-beam strip, shown in Fig. 1, are Eqs. (1 and 2).

Figure 1: Flat plate slab-beam strip



$$M_N = \frac{EI_{SB}}{L_1}(S_{SB}\,\theta_N + Sc_{SB}\,\theta_F) \qquad (1)$$

$$M_F = \frac{EI_{SB}}{L_1}(Sc_{SB}\,\theta_N + S_{SB}\,\theta_F) \qquad (2)$$

Where, S_{SB} and Sc_{SB} are slab-beam stiffness coefficients, these stiffness coefficients are calculated by fitting data of ACI319-12 [1]; S_{SB} and Sc_{SB} are given by Eqs. (3 and 4).

$$S_{SB} = 4 - \left(25\,R_1^2 - 23.25\,R_1 + \frac{9}{40}\right)R_2 + \left(87.5\,R_1^2 - 15R_1 + \frac{3}{8}\right)R_2^2 \qquad (3)$$

$$Sc_{SB} = 2 - (31.8\,R_1^2 - 22.43\,R_1 + 0.715)R_2 + \left(110.7\,R_1^2 - 27.26R_1 + \frac{9}{4}\right)R_2^2 \qquad (4)$$

In Eq.s. (3 and 4), it's assumed that $(CN_1 = CF_1 = C_1$ and $CN_2 = CF_2 = C_2)$. $R_1 = C_1/L_1$ and $R_2 = C_2/L_2$. Table 1 shows data comparisons of Table A1 [1] and Eq.s. (3 and 4).Also, it shows high accuracy have been achieved between [1]and Eq.s. (3 and 4). I_{SB} is a moment of inertia of portion of slab-beam strip which resist and transfer the moments from slab to column according to GB 50010-2010 [19] given by:

$$I_{SB} = \frac{b_{eff}\,t^3}{12} \qquad (5)$$

b_{eff} is the effective slab width which considered effective for resisting unbalanced moments transferred between the slab and column as defined by the [20] ,[30] and according clause 6.6.2 of [19] its determination is as follow:

For interior column

$$b_{eff} = C_2 + 3\,t \qquad (6)$$

For exterior column

$$b_{eff} = C_2 + 1.5\,t \qquad (7)$$

Where t is slab thickness in (mm) to be greater than or equal (L1 - C1) / 30.

Table 1: Data comparisons data of ACI318-12 [1] and Eq.s. (3 and 4).

R_1	R_2	k_{NF}	S_{BB} Eq. (3)	k_{NF}, C_{NF}	Sc_{SB} Eq. (4)

0	-	4.00	4.00	2.00	2.00
0.10	0.00	4.00	4.00	2.00	2.00
	0.10	4.18	4.18	2.13	2.13
	0.20	4.36	4.36	2.27	2.27
	0.30	4.53	4.53	2.44	2.42
	0.40	4.70	4.70	2.59	2.59
0.20	0.00	4.00	4.00	2.00	2.00
	0.10	4.35	4.35	2.26	2.26
	0.20	4.72	4.72	2.55	2.55
	0.30	5.11	5.11	2.86	2.86
	0.40	5.51	5.51	3.19	3.19
0.30	0.00	4.00	4.00	2.00	2.00
	0.10	4.49	4.49	2.38	2.36
	0.20	5.05	5.05	2.83	2.79
	0.30	5.69	5.6875	3.36	3.31
	0.40	6.41	6.4	3.91	3.91
0.40	0.00	4.00	4	2.00	2.00
	0.10	4.61	4.59	2.44	2.41
	0.20	5.35	5.35	3.00	3.00
	0.30	6.25	6.276	3.75	3.77
	0.40	7.37	7.37	4.72	4.72

3. Column stiffness

By benefit from the data provided in [1], the column in flat plate stiffness coefficients (S_0 and Sc_0) are illustrated in Eq.s. (8 and 9), these equation are extracted by fitting the data of [1].

$$S_0 = 4\,(1 + 2.38\,R_3 + 2.86\,R_3^2) \qquad (8)$$

$$Sc_0 = 2(1 + 3.68\,R_3 + 5.82\,R_3^2) \qquad (9)$$

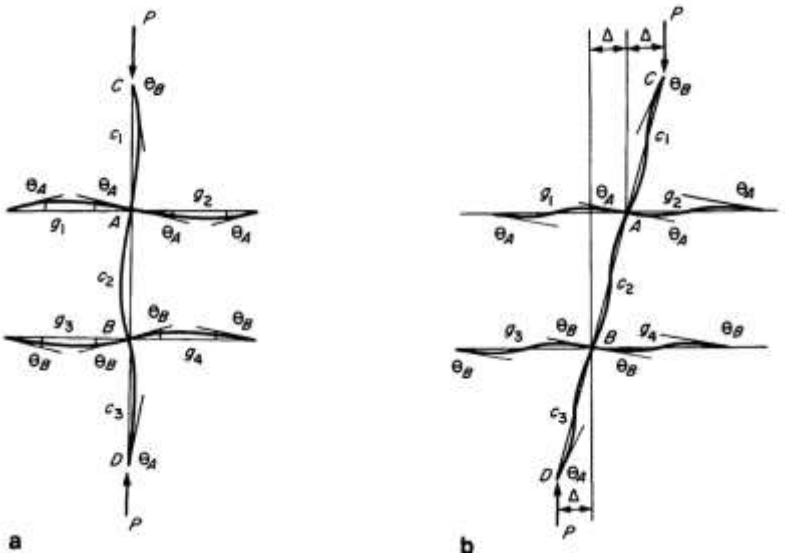
Where R_3= t / H_c, H_c is column unsupported length, in Eq.s. (8 and 9), the upper and lower slabs are of same thickness, t. Fitting accuracy checking of Eq.s. (8 and 9) is shown in Table 2 [1] use (ta/tb=1 and H/Hc=1+ R_3) or R_3= H/Hc -1.

Table 2: Data comparisons of ACI318-12 [1] data and Eq. (8 and 9).

R_3	(kAB)	S_0 Eq. (8)	(kAB. CAB)	Sc_0 Eq. (9)
0.05	4.52	4.51	2.44	2.40
0.10	5.09	5.07	2.90	2.85
0.15	5.71	5.69	3.43	3.37
0.20	6.38	6.36	3.96	3.94
0.25	7.11	7.10	4.62	4.57
0.30	7.89	7.89	5.29	5.26
0.35	8.73	8.73	6.02	6.00
0.40	9.63	9.64	6.84	6.81
0.45	10.60	10.60	7.74	7.67
0.50	11.62	11.62	8.59	8.59

In braced and unbraced flat plate system, the closed form equation of effective length factor are obtain using various approaches K-factor approximate approach, column stiffness modification approach), these approaches adopted subassembly model shown in Fig. 2 and the value of K-factors are evaluated using Newton-Raphson iteration, these methods are illustrated in below.

Figure 2: Subassembly models of K-factors (closed form derivation).
(a) Braced frame, (b) Unbraced frame.



4. Braced frame

By referring to Fig. 2 (a), it is observed that the subassembly frame contain two degrees of freedom ($\theta_A$and θ_B), the resulted stiffness determinant is 2×2. Two stability analysis sets of K-factor derivation are investigated in below.

K-factor approximate approach

Referring to column stiffness equations (8 and 9), and by ignoring the influence of slab portion considering $R_3 = 0$, the second-order column stiffness equations write as[8], [10]:

$$S_K = \frac{\left(\frac{\pi}{K}\right)\sin\left(\frac{\pi}{K}\right) - \left(\frac{\pi}{K}\right)^2\cos\left(\frac{\pi}{K}\right)}{2 - 2\cos\left(\frac{\pi}{K}\right) - \left(\frac{\pi}{K}\right)\sin\left(\frac{\pi}{K}\right)} \qquad (10)$$

$$Sc_K = \frac{\left(\frac{\pi}{K}\right)^2 - \left(\frac{\pi}{K}\right)\sin\left(\frac{\pi}{K}\right)}{2 - 2\cos\left(\frac{\pi}{K}\right) - \left(\frac{\pi}{K}\right)\sin\left(\frac{\pi}{K}\right)} \qquad (11)$$

Applying slop-deflection equations considering braced subassembly model and incorporated slab-beam stiffness eqs. (3 and 4). The following system buckling relation is achieved

$$f(K) = \begin{vmatrix} GBSc_K & \beta_B + GB\,S_K \\ \beta_A + GA\,S_K & GA\,Sc_K \end{vmatrix} = 0 \qquad (12)$$

To evaluate K value, solution of Eq. (12) is required using iterations; Newton-Raphson iterative type is adopted and illustrated as in the following steps.

$$f(K) = \frac{\beta_A\beta_B}{GA\,GB} + S_K\left(\frac{\beta_A}{GA} + \frac{\beta_B}{GB}\right) + {S_K}^2 - {Sc_K}^2 \qquad (13)$$

$$f'(K) = {S'}_K\left(\frac{\beta_A}{GA} + \frac{\beta_B}{GB}\right) + 2\,{S'}_K S_K - 2\,{Sc'}_K Sc_K \qquad (14)$$

$$K_{n+1} = K_n - \frac{f(K_n)}{f'(K_n)} \qquad (15)$$

Continue until $|f(K)| \leq$ toleranc.

Where

$$\beta = S_{SB} - Sc_{SB} \qquad (16)$$

$$G = \frac{\sum(EI/L)_C}{\sum(EI/L)_{SB}} \qquad (17)$$

$${S'}_K = \frac{(K^2(S_K^2 - S_K) + \pi^2 Sc_K)\sin\left(\frac{\pi}{K}\right) - \pi\,K\,S_K\,(S_K - 1)}{K^2\left(K\sin\left(\frac{\pi}{K}\right) - \pi\right)} \qquad (18)$$

$${Sc'}_K = \frac{(S_K - 2)Sc_K - S_K}{K} \qquad (19)$$

Depending on this solution type and incorporate with Newmark formula Dumonteil [13], the following solution approximate equation of K determination is proposed, eq. (20).

$$K = \sqrt{\frac{[GA + 0.2\,\beta_A][GB + 0.2\,\beta_B]}{[GA + 0.4\,\beta_A][GB + 0.4\,\beta_B]}} \qquad (20)$$

Eq. (20) is modified approximate K-factor equation of column in braced frame of the first solution approach.

5. Column stiffness modification approach

This solution approach is considered as modified solution strategy, the second order column stiffness equations can be obtained by linearly improving Eq.s. (8 and 9) by replace the linear stiffness coefficients, 4 and 2, by column second-order stability functions Al-Ghalibi [4] to be written as

$$S = S_K \, Go_1 \qquad (21)$$

$$Sc = Sc_K \, Go_2 \qquad (22)$$

Where, Go_1 and Go_2 are column stiffness geometry functions, Eq.s. (23 and 24).

$$Go_1 = 1 + 2.38 \, R_3 + 2.86 \, R_3^2 \qquad (23)$$

$$Go_2 = 1 + 3.68 \, R_3 + 5.82 \, R_3^2 \qquad (24)$$

Slop-deflection equations are resulted the following buckling relation

$$f(K) = \begin{vmatrix} GB \, Go_2 Sc_K & \beta_B + GB \, Go_1 S_K \\ \beta_A + GA \, Go_1 S_K & GA \, Go_2 Sc_K \end{vmatrix} = 0 \qquad (25)$$

By expand Eq. (25), Newton-Raphson iterative solution required that

$$f(K) = \frac{\beta_A \, \beta_B}{GA \, GB} + Go_1 S_K \left(\frac{\beta_A}{GA} + \frac{\beta_B}{GB} \right) + {Go_1}^2 \, {S_K}^2 - {Go_2}^2 \, {Sc_K}^2 \qquad (26)$$

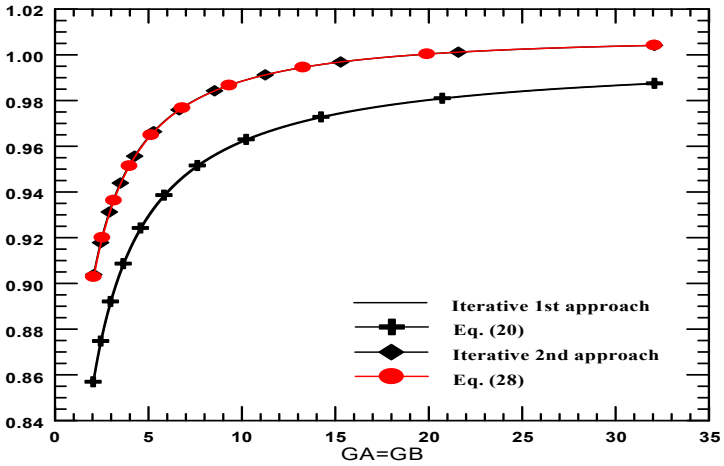
$$f'(K) = Go_1 S'_K \left(\frac{\beta_A}{GA} + \frac{\beta_B}{GB} \right) + 2 \, {Go_1}^2 \, S'_K \, S_K - 2 \, {Go_2}^2 \, Sc'_K \, Sc_K \qquad (27)$$

The proposed K-factor equation adopted this solution type is investigated by improving Newmark equation Dumonteil [12] as suggested by Eq. (28).

$$K = \sqrt{\frac{Go_2[GA + 0.19 \, \beta_A][GB + 0.19 \, \beta_B]}{Go_1[GA + 0.38 \, \beta_A][GB + 0.38 \, \beta_B]}} \qquad (28)$$

For the purpose of comparing the iterative solution method and the proposed approximate equation approach, both solution methods were studied to calculate the effective length factor of a column in braced flat plate system with the following specifications: Column dimensions (C1=350 mm, C2= 400 mm, Hc=3.5 m), Frame dimensions (L1 (3m – 8m), L2=5 m), GA=GB. Fig. 3 below illustrates the relationship between the variation of the G parameter for the column and the K factor. In general, the graph demonstrates the high accuracy of the proposed approximate Eq.s. (20 and 28).

Figure 3: G-parameter and K-factors relations of braced column of both solution approaches.



6. Unbraced frame

In case of sway allowed flat plate system, same previous two solution approaches are adopted for subassembly model shown in Fig. 2(b). The subassembly frame contains three degrees of freedom ($\theta_A, \theta_B,$ and Δ), the resulted stiffness determinant is 3×3. Two stability analysis sets of K-factor derivation are explored as in the following.

7. K-factor approximate approach

Referring to column stiffness equations (8 and 9), by neglecting the influence of slab contribution term, $R_3 = 0$, the second-order column stiffness equations are (10 and 11). The buckling determinant of the unbraced subassembly model can be obtaining using slop-deflection equations including the influence of slab-beam stiffness Eqs. (3 and 4), this determinant is Eq. (29).

$$f(K)=\begin{vmatrix} Sc_K & \frac{\beta_B}{GB}+S_K & S_K+Sc_K \\ \frac{\beta_A}{GA}+S_K & Sc_K & S_K+Sc_K \\ S_K+Sc_K & S_K+Sc_K & 2S_K+2Sc_K-\frac{\pi^2}{K^2} \end{vmatrix}=0 \qquad (29)$$

Expand Eq. (29) resulted (30).

$$f(K)=\left[\frac{\beta_A}{GA}+\frac{\beta_B}{GB}\right]\left(Sc_K^2-S_K^2+S_K\frac{\pi^2}{K^2}\right)+\frac{3[\beta_A+\beta_B]}{GA\ GB}\left(\frac{\pi^2}{K^2}-2S_K-2Sc_K\right)+\frac{\pi^2}{K^2}(S_K^2-Sc_K^2) \qquad (30)$$

The derivative of (30) is (31).

$$f'(K)=\left[\frac{\beta_A}{GA}+\frac{\beta_B}{GB}\right]\left(2\,Sc_K\,Sc_K'-2\,S_K\,S_K'-\frac{2\,\pi^2}{K^3}S_K+\frac{\pi^2}{K^2}S_K'\right)-\frac{6\,[\beta_A+\beta_B]}{GA\ GB}\left(\frac{\pi^2}{K^3}+S_K'+Sc_K'\right)$$
$$+\frac{\pi^2}{K^2}(2\,S_K\,S_K'-2\,Sc_K\,Sc_K')-\frac{2\,\pi^2}{K^3}(S_K^2-Sc_K^2) \qquad (31)$$

Where

$$\beta=S_{SB}+Sc_{SB} \qquad (32)$$

To evaluate the closed value of K-factor use Eqs.(30 and 31) with Newton-Raphson iterations. By modify French "CM66" equation for unbraced frame Dumonteil [13], the improved French formula of approximate K-factor equation is (33).

$$K=\sqrt{\frac{57.6\ GA\ GB}{\beta_A\,\beta_B}+24\left(\frac{GA}{\beta_A}+\frac{GB}{\beta_B}\right)+20\ \Big/\ 6\left(\frac{GA}{\beta_A}+\frac{GB}{\beta_B}\right)+20} \qquad (33)$$

8. Column stiffness modification approach

The second order column stiffness equations are obtained by linearly modifying of Eqs. (8 and 9), replacing the linear stiffness parameters, 4 and 2, in Eqs. (8 and 9) by column second-order stability functions Eqs. (10 and 11) Al-Ghalibi [4]as explained in Eqs. (21 and 22). Apply slop-deflection equations to the subassembly sway-allowed frame of Fig. 2 (b), the following stiffness-buckling relation is achieved.

$$f(K)=\begin{vmatrix} Go_2Sc_K & \frac{\beta_B}{GB}+Go_1\,S_K & Go_1S_K+Go_2Sc_K \\ \frac{\beta_A}{GA}+Go_1S_K & Go_2Sc_K & Go_1S_K+Go_2Sc_K \\ Go_1S_K+Go_2Sc_K & Go_1S_K+Go_2Sc_K & 2Go_1S_K+2Go_2Sc_K-\frac{\pi^2}{K^2} \end{vmatrix}=0 \qquad (34)$$

Eq. (35) is yield after expand the determinant of Eq. (34).

$$f(K)=\left[\frac{\beta_A}{GA}+\frac{\beta_B}{GB}\right]\left(Go_2^2Sc_K^2-Go_1^2S_K^2+Go_1S_K\frac{\pi^2}{K^2}\right)+\frac{3[\beta_A+\beta_B]}{GA\ GB}\left(\frac{\pi^2}{K^2}-2Go_1S_K-2Go_2Sc_K\right)$$
$$+\frac{\pi^2}{K^2}(Go_1^2S_K^2-Go_2^2Sc_K^2) \qquad (35)$$

The first derivative of (35) is (36).

$$f'(K)=\left[\frac{\beta_A}{GA}+\frac{\beta_B}{GB}\right]\left(2\,Go_2^2\,Sc_K\,Sc_K'-2\,Go_1^2S_K\,S_K'-\frac{2\,\pi^2}{K^3}Go_1\,S_K+\frac{\pi^2}{K^2}Go_1S_K'\right)$$
$$-\frac{6\,[\beta_A+\beta_B]}{GA\ GB}\left(\frac{\pi^2}{K^3}+Go_1S_K'+Go_2Sc_K'\right)+\frac{\pi^2}{K^2}(2\,Go_1^2\,S_K\,S_K'-2\,Go_2^2\,Sc_K\,Sc_K')$$
$$-\frac{2\,\pi^2}{K^3}(Go_1^2\,S_K^2-Go_2^2\,Sc_K^2) \qquad (36)$$

Iterative required solving Eq. (35) to evaluate K-factor using Newton-Rahson iterative type. French "CM66" equation of unbraced frame Dumonteil [13] is improved as in Eq. (37), the improved French formula of approximate K-factor Eq. (37) is considered as approximate solution of Eq. (35).

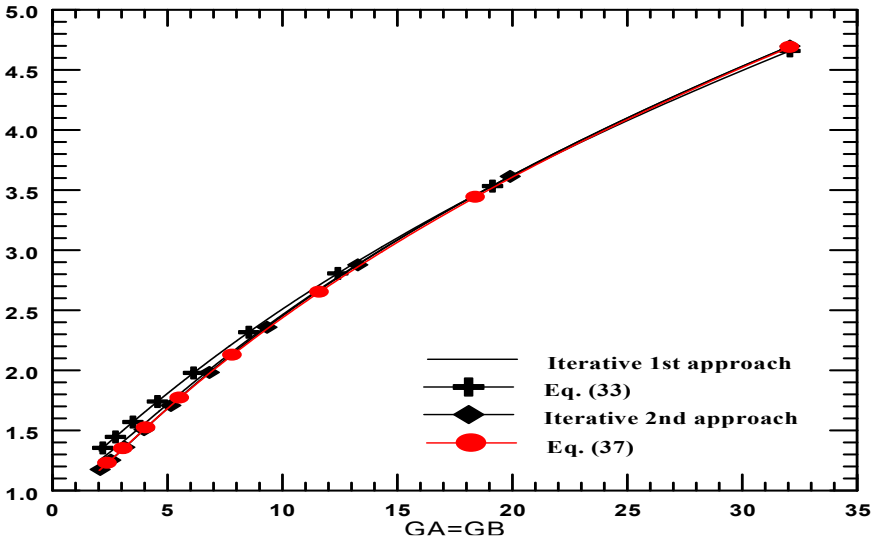
$$K=\sqrt{\varphi\left[\frac{57.6\ GA\ GB}{\beta_A\beta_B}+24\left(\frac{GA}{\beta_A}+\frac{GB}{\beta_B}\right)+20\right]\ \Big/\ 6\left(\frac{GA}{\beta_A}+\frac{GB}{\beta_B}\right)+20} \qquad (37)$$

$$\varphi=0.94+10.31\,R_3-357.42\,R_3^2+2528.43\,R_3^3 \qquad (38)$$

For a column in a laterally unsupported frame, a comparison was conducted between the iterative solution method and the proposed approximate equations for the two previously mentioned solution approaches. A column with the

following specifications was studied: Column dimensions (C1=350 mm, C2= 400 mm, Hc=3.5 m), frame dimensions (L1 (3m – 8m), L2=5 m), GA=GB. Fig. 4 demonstrates the high accuracy of the proposed approximate equations compared to the iterative method. Thus, it can be concluded that the approximate equations suggested in the current study are reliable for calculating the effective length factor of a column in a laterally unsupported building frame system.

Figure 4: G-parameter and K-factors relations of unbraced column of both solution approaches.



9. Modification to the alignment charts of current US National codes

For practical use, the present second solution approach is employed to develop current alignment charts of US National codes shown in Fig. 5 (a and b) by follow the following procedure to dealing with column in braced and unbraced flat plate.

a. For braced frame

Evaluate the modified G parameter at column ends A and B

$$G = \frac{\sum(EI/L)_C}{\sum \gamma(EI/L)_{SB}} \qquad (39)$$

γ is slab-beam stiffness modification parameter given by Eq. (40) Al-Ghalibi **[16]**.

$$\gamma = \frac{\beta}{2} \qquad (40)$$

$$K = m\, K_{\text{alignment chart}} \qquad (41)$$

$$m = 1 + 0.6\, R_3 + 1.8\, R_3^2 \qquad (42)$$

b. For unbraced frame

Calculate modified G parameter using Eq. (39), the stiffness modification factor, γ, is Eq. (43) Al-Ghalibi **[16]**.

$$\gamma = \frac{\beta}{6} \qquad (43)$$

$$K = m\, K_{\text{alignment chart}} \qquad (44)$$

$$m = 0.61 + 0.14\, \omega - 0.016\, \omega^2 \qquad (45)$$

And

$$\omega = 10\, R_3 \frac{GA\ GB}{GA + GB} \qquad (46)$$

Figure 5: Alignment charts of K-factor determination (US National codes) Braced frame, (b) Unbraced frame.

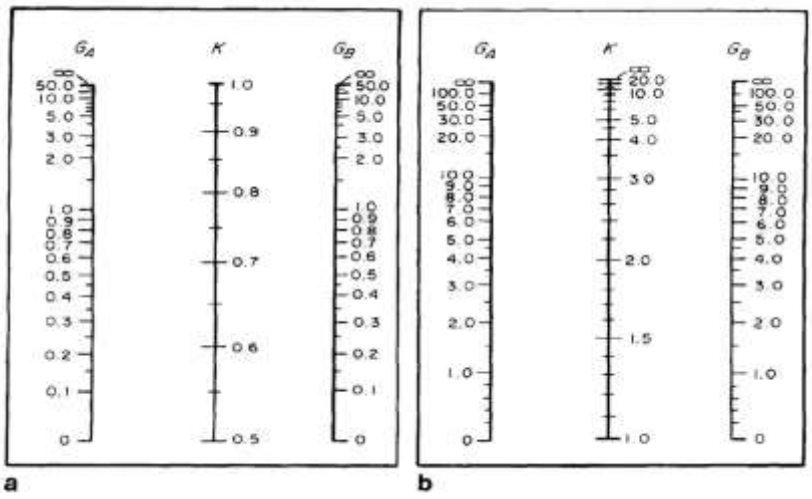


Figure 6: K-factor of present study and world codes and studies of column in braced flat plate.

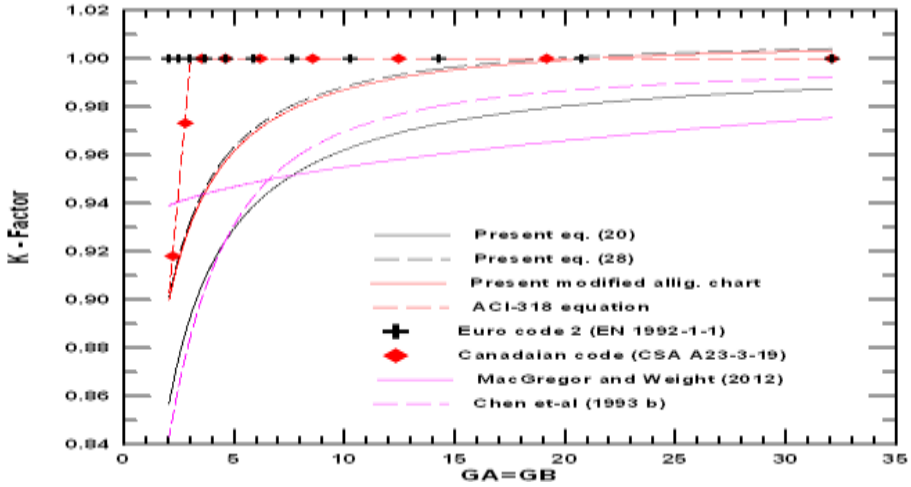
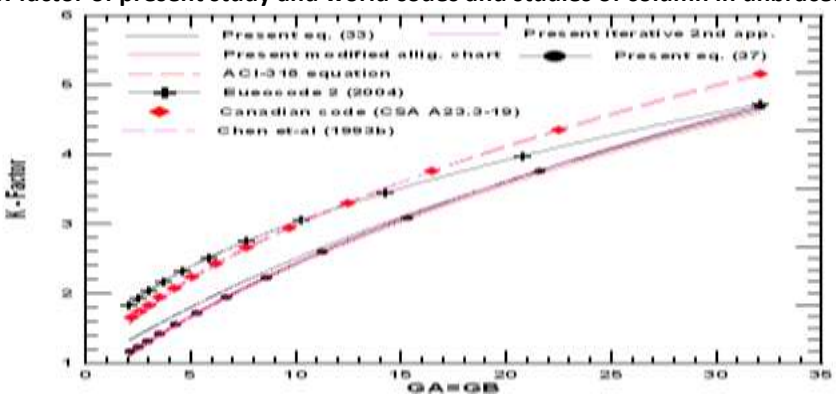


Figure 7: K-factor of present study and world codes and studies of column in unbraced flat plate.



Intensive world specifications and researches comparisons with the present study equations of column in braced and unbraced frame are shown in Figs. 6 and 7. A column with the following properties was studied: Column dimensions (C1=350 mm, C2= 400 mm, Hc=3.5 m), frame dimensions (L1 (3m – 8m), L2=5 m), GA=GB.

Fig. 6 presents a comparison of the K-factor values for the studied column in braced frame of the aforementioned characteristics, an excellent agreement is observed between Eq. 28 and the results of the developed alignment chart of current US National codes method adopted in the current study. All the referenced studies and specifications used for comparison neglect the influence of the R1, R2, and R3 coefficients, these three coefficients have been incorporated into the current study, and their impact on the stability functions has been included; neglecting these coefficients leads to inaccurate K-factor calculations. As shown in Fig. 6, the results of the second solution approach and the proposed development alignment chart method converge with those of the American Concrete Institute equation ACI318-12 [1], EN 1992-1-1 [20], and the CSA [22] standards when GA = GB > 20. The comparative results

shown in Fig. 7 for a column in an unbraced frame demonstrate a strong agreement in the calculated K-factor values among the three proposed methods of the current study, all of which account for the coefficients R1, R2, and R3. Due to ignoring the influence of R1, R2, and R3 factors, Fig. 7 shows reveals deviation from realistic results of [18] CSA [22], Chen et-al [15] equation, EN 1992-1-1 [20]. The aforementioned specifications and studies need to incorporate the influence of these three coefficients (R1, R2, and R3) and perform a second-order analysis, as demonstrated in the current study, to accurately calculate the effective length factor of a column in braced and unbraced flat plate system.

10. Illustrative examples

Example 1: An interior column in second story braced flat plate its dimensions (C1= 400 mm, C2= 300 mm), the slab dimensions (L1= 7000 mm, L2= 6000 mm), the column clear height (Hc= 3000 mm). Assume slab thickness (t= 250 mm). Determine K-factor for the column.

Solution

By first solution approach

Hc= 3000 mm.

R1= 0.057, R2= 0.05, R3=0.0834.

Ic= 16×10^8 mm^4, beff= 1050 mm, Isb= 13.672×10^8 mm^4, GA= GB= 2.73, $\beta_A = \beta_B = 2.025$

By iterative, K=0.884, Using Eq. (20), K=0.885.

By second solution approach

Hc= 3000 mm.

R1= 0.057, R2= 0.05, R3= 0.0834.

Ic= 16×10^8 mm^4, beff= 1050 mm, Isb= 13.672×10^8 mm^4, GA= GB= 2.73, $\beta_A = \beta_B = 2.025$

By iterative, K= 0.9397, Using Eq. (28), K= 0.936.

By modification to the alignment chart method (Hc=3000 mm), R3= 0.0834

Using Eq. (40), $\gamma = 1.0125$, modified GA= GB= 2.73/ 1.0125=2.7 use alignment chart, Fig. 5 (a), k= 0.88, Using Eq. (42), $m = 1.0625$, K= 1.0625× k = 0.935

According to ACI 318 [19]:

$$K = 0.7 + 0.05(GA + GB) \leq 1$$
$$K = 0.7 + 0.05(2.73 + 2.73) = 0.973$$

Or

$$K = 0.85 + 0.05\, G_{min} \leq 1$$
$$K = 0.85 + 0.05 \times 2.73 = 0.9856$$

Use K=0.973

According to EN 1992-1-1 [19], GB 50010-2010 [17], and IS 456:2000 [21]:

$$K = 0.5 + 0.14(GA + GB) + 0.055(GA + GB)^2 \leq 1$$
$$K = 0.5 + 0.14(2.73 + 2.73) + 0.055(2.73 + 2.73)^2 = 2.9 > 1$$
$$K = 1$$

According to CSA [22].

$$K = 0.7 + 0.05(GA + GB) \leq 1$$
$$K = 0.7 + 0.05(2.73 + 2.73) = 0.973$$

By using Simplified Empirical Formula: MacGregor and Wight [23].

$$K = 0.5 + 0.014\left(\frac{L_1}{t}\right)$$
$$K = 0.5 + 0.014\left(\frac{7000}{250}\right) = 0.892$$

Chen et al. [14]

$$K = 1 - \frac{1}{5 + 9\,GA} - \frac{1}{5 + 9\,GB} - \frac{1}{10 + GA\,GB}$$
$$K = 1 - \frac{1}{5 + 9 \times 2.73} - \frac{1}{5 + 9 \times 2.73} - \frac{1}{10 + 2.73 \times 2.73} = 0.875$$

Table 3: Results summary of Example 1

Study	K
Present study- first approach, iterative	0.88
Present study- first approach, Eq. (20)	0.89
Present study- second approach, iterative	0.93
Present study- second approach, Eq. (28)	0.94
Present study- modified alignment chart	0.94
ACI 318 [18]	0.97
EN 1992-1-1 [19]	1.00
GB 50010-2010 [17]	1.00
IS 456-2000[**20]**	1.00
C**SA** [21]	0.97
MacGregor and Wight [22]	0.89
Chen et al. [14]	0.88

Example 2: Consider same column in unbraced frame.
By first solution approach
Hc= 3000 mm
R1= 0.057, R2= 0.05, R3= 0.0834
Ic= 16 $\times 10^8$ mm^4, beff= 1050 mm, Isb= 13.672$\times 10^8$ mm^4, GA= GB= 2.73, $\beta_A = \beta_B = 6.07$
By iterative, K=1.357, Using Eq. (33), K=1.45
By second solution approach.
Hc= 3000 mm
R1= 0.057, R2= 0.05, R3= 0.0834
Ic= 16 $\times 10^8$ mm^4, beff= 1050 mm, Isb= 13.672$\times 10^8$ mm^4, GA= GB= 2.73, $\beta_A = \beta_B = 2.025$
By iterative, K= 1.27, Using Eq. (37), K= 1.28

By modification to the alignment chart method (Hc=3000 mm) , R3= 0.0834
Using Eq. (43), $\gamma = 1.01$
Modified GA= GB= 2.73/1.01=2.7 use alignment chart, Fig. 5 (b), k= 1.73
Using Eq. (45), $m = 0.747$
K= 0.747 $\times$ k= 1.29

According to ACI-318 [20].

$$G_m = 2.73 > 2$$
$$K = 0.9\sqrt{1 + G_m}$$
$$K = 0.9\sqrt{1 + 2.73} = 1.738$$

For EN 1992-1-1 [21], Stiffness Ratio-Based Formulas:.

$$K = \sqrt{1 + 10\,\frac{k_1 k_2}{k_1 + k_2}} \geq \left(1 + \frac{k_1}{k_1 + 1}\right)\left(1 + \frac{k_2}{k_2 + 1}\right)$$

$$k_1 = \frac{(EI/L)_C}{(EI/L)_S} = \frac{16 \times 10^8/3000}{\frac{1}{12} \times 250^3 \times 6000/7000} = 0.477$$

$$k_2 = k_1$$

$$K = \sqrt{1 + 10\,\frac{0.477 \times 0.477}{0.477 + 0.477}} \geq \left(1 + \frac{0.477}{0.477 + 1}\right)\left(1 + \frac{0.477}{0.477 + 1}\right)$$

$$K = 1.87 \geq 1.75$$

K=1.87
According to GB 50010-2010 [18], and CSA [22].

$$K = \sqrt{\frac{1.6\,GA\,GB + 4(GA + GB) + 7.5}{GA + GB + 7.5}}$$

$$K = \sqrt{\frac{1.6 \times 2.73 \times 2.73 + 4(2.73 + 2.73) + 7.5}{2.73 + 2.73 + 7.5}} = 1.784$$

Chen et al. [14]

$$K = 4 - \frac{1}{1 + 0.2\,GA} - \frac{1}{1 + 0.2\,GB} - \frac{1}{1 + 0.01\,GA\,GB}$$

$$K = 4 - \frac{1}{1 + 0.2 \times 2.73} - \frac{1}{1 + 0.2 \times 2.73} - \frac{1}{1 + 0.01 \times 2.73 \times 2.73} = 1.7757$$

Table 4: Results summary of Example 2

Study	K
Present study- first approach, iterative	1.36
Present study- first approach, Eq. (33)	1.45
Present study- second approach, iterative	1.27
Present study- second approach, Eq. (37)	1.28
Present study- modified alignment chart	1.29
ACI 318 [18]	1.74
EN 1992-1-1 [19]	1.84
GB 50010-2010 [17]	1.78
CSA [21]	1.78
Chen et al. [14]	1.78

11. Results discussions

The results of the first example demonstrate that the effective length factor of the column calculated in the current study closely aligns with the values stipulated in the EN 1992-1-1 [21], GB 50010-2010 [18], and IS 456-2000 [22] standards, while showing good agreement with the ACI 318 [20] and CSA [23] standard at a 3.9 % upper difference. The results of the first example also show a lower difference of 4.6% compared to the study conducted by MacGregor and Whght [24] of a 6.4% lower difference from the study presented by Chen et al. [15]. Regarding the second example, most internationally recognized standards and studies yield relatively higher results compared to the values calculated in the current study, this discrepancy in some results stems from neglecting the influence of the stiffness coefficients R1, R2, and R3, as well as their exclusion from the stability functions during the second-order analysis. For both examples analyzed in this study, the method of developing the original G coefficient—adopted in the current US National standards and proposed in this study—can be considered as a simple and effective approach for calculating the effective length factor for column in braced and unbraced flat plate system.

12. Conclusions

The present study introduced the calculation of effective length factors for column in braced and unbraced flat slab system. Linking stiffness parameters data of EFM with second order analysis are presented, slab-beam and column stiffness are formulated depending on the data provided in Notes of ACI-318 [19] code. Two solution approaches are adopted and suggestion of approximate equations for each approach is presented. Depending on simple enhancement of the original G parameter using stiffness modification factor, γ, evaluation of improved K-factor incorporated with the current US National codes alignment charts, have been investigated for both braced and unbraced columns. Two examples are given to numerically clarify the use of the suggested methods procedure, it's observed that the proposed approximate equations provided high accuracy as compared with closed form solution using iterative, also, the current US National code modification method using alignment charts can be considered as simple and practical method of K-factors evaluation of column of flat slab system. Examples results of the present study are compared with K-factor evaluation of many world specifications.

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